Math 270: Differential Equations Day 10 Part 1

Section 4.3: Solving Second-Order Linear Homogeneous DEs With Constant Coefficients Where The Auxiliary Equation Has Complex Roots

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In Section 4.2, we learned how to solve second-order linear homogeneous DEs with constant coefficients

$$ay'' + by' + cy = 0$$

where the auxiliary equation

$$ar^2 + br + c = 0$$

only had real roots.

<u>Question</u>: What if the auxiliary equation has complex roots?

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<u>Question</u>: What if the auxiliary equation has complex roots?

Discuss

- If r is a root to aux eqn then $y = e^{rx}$ is a solution to the DE (even if r is complex)
- $e^{i\theta}$
- Use fact that solutions are a vector space (over $\mathbb{C}$) but we want real solutions

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Question: What if the auxiliary equation has complex roots?
Answer: If $\alpha + i\beta$ is a complex root of the auxiliary equation, then $y_1 = e^{\alpha x} \sin \beta x$ and $y_2 = e^{\alpha x} \cos \beta x$ are two independent solutions to the DE

Ex 1: Suppose $3 + 5i$ is a root of the auxiliary equation. Find a pair of independent solutions to the DE.

Note: Complex roots come in conjugate pairs, but when applying this result for a complex root, you can ignore it's conjugate.

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Ex 2: Find all solutions to $y'' + 4y = 0$

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Ex 3: Find all solutions to $2y'' - 3y' + 8y = 0$

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Ex 4: A second-order linear homogeneous DE with constant coefficients roots listed in the following table. Find the general solution to the DE.

Root	Multiplicity
2	3
-3	1
4i	1
-4i	1
5-2i	2
5+2i	2

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Ex 5: Solve the IVP

$$y''' - 4y'' + 7y' - 6y = 0; \quad y(0) = 1\,, \quad y'(0) = 0\,, \quad y''(0) = 0$$